Relative Gasoline Prices: Octanes and Arbitrage

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ABSTRACT

The mixing of gasolines produces a blend having an octane that is a weighted average of the components' octanes. Applying Modigliani and Miller's (1958) logic regarding milk markets and capital structures, gasoline prices should be linear in the octane mixing fraction. We document that such pricing closely describes wholesale –but not retail– gasoline prices. An enduring and robust observation about retail prices is that the posted midgrade price is above the apparent cost of producing it by mixing regular and midgrade. While it may be appropriate for milk markets and capital structures, retail midgrade gasoline pricing demonstrates that the wisdom of Modigliani and Miller's (1958) relying "merely on the fact that a given commodity cannot consistently sell at more than one price in the market" is not clearly established.

Introducing a new product presents a pricing challenge for wholesalers and retailers alike. In the market for a firm's *financial* products (its stocks and bonds), spanning, replication and the possibility of arbitrage trading suggest equilibrium pricing restrictions that play a central role in theories of how those financial products are priced. In contrast, spanning, replication and arbitrage trading are seldom, if ever, central considerations in theories leading to restrictions on how that same firm's *nonfinancial* products are priced. While it is true that many nonfinancial product markets inherently preclude the type of post-production (post-issuance) consumer behavior assumed in financial market pricing analyses, this need not always be the case.

In this paper we investigate how a new --yet spanned-- product is priced in what many would characterize as a market dominated by price competition. The gasoline market is an example of a nonfinancial product market where buyers can easily engage in low-cost post-production actions leading to asset spanning, replication and potential arbitrage opportunities. We focus on the resulting equilibrium pricing restrictions and on the empirical case for those restrictions in the wholesale and retail segments of that nonfinancial product market.

Financial theory almost uniformly adopts the notion that buyers of financial assets employ a post-production approximately-linear mixing technology (portfolio formation). Indeed, hypothetical access to a costless post-production (post-issue) approximately-linear mixing technology is the *deus ex machina* for much of what we understand about capital structure theory (costless portfolio formation involving securities from the same firm) and capital asset pricing theory (costless portfolio formation involving securities from different firms). In fact, much of modern financial theory derives from the notion that the formation and trading of security portfolios should not provide a "free lunch"; equilibrium asset prices should preclude arbitrage opportunities. In introducing arbitrage considerations to financial markets**,** Modigliani and Miller (1958) suggest that arbitrage restrictions in financial markets are reasonable precisely because they are reasonable in nonfinancial commodity markets, in particular the wholesale and retail milk and butter fat markets. They remark that their results:

> "rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be

> consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle)."

Upon invoking the milk market analogy, Modigliani and Miller state that their capital structure conclusions are:

> "equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk."

Their more formal argument (in a footnote) is that milk prices are linear in the mixing fraction and that total receipts are independent of the skimming decision.

In the consumer gasoline market, buyers and sellers have access to a similar low-cost post-production approximately-linear mixing technology. With some restrictions, market participants can mix grades of octane and replicate a subset of existing product offerings. This suggests that concepts commonly used to analyze financial products may be useful when looking at the gasoline market. We draw analogies from financial market theories and consider the pricing ramifications of octane spanning, octane replication and octane arbitrage trading.

In reality, transactions costs and other frictions can inhibit the convergence of prices to their hypothetical arbitrage-free levels. When transactions costs are low, however, we would expect convergence at a relatively rapid rate. This is true for financial and non-financial product markets. Our examination of gasoline pricing is in part an attempt to extend the reach of arbitrage pricing to nonfinancial commodity markets offering post-purchase linear mixing (where some of the motivating intuition originated). We also see the examination of retail gasoline pricing as an opportunity to revisit the widely accepted notion underlying modern capital structure theory and other modern financial theories that a necessary condition for equilibrium is the absence of arbitrage opportunities.

We examine a product market that is somewhat unique in the ease by which a menu of products can be repackaged. Specifically, gasolines can be mixed by distributors at the rack (a common practice) or by consumers at the pump (a much less common practice). Such mixing can produce intermediate octane products that are perfect substitutes for those offered directly by retailers and wholesalers. We find gross violations of arbitrage-free pricing at the retail gasoline pump but not at the wholesale rack. Specifically, linear combinations of posted regular and premium gasoline prices conform closely to posted midgrade prices at the wholesale level, but deviate substantially at the retail level. Our evidence shows that this mispricing exists in aggregate U.S. data and in individual retail station data. It has persisted during the entire period of our study, 1996 to 2006. As an illustration of the magnitude of this mispricing, consumer mixing at the pump to create retail midgrade could have saved consumers over $1 billion between 1996 and 2006.

1. Octane Spanning, Replication and Arbitrage

In the mid 1980s, retail gasoline stations began selling three octane grades of unleaded gasoline: regular, midgrade, and premium. According to industry watchers, one rationale for midgrade's introduction was to provide a more complete menu of octane offerings that would allow additional customers to upgrade their octane and potentially increase premium's separation – in perception and price – from regular.[1] While the vast majority of stations offer three grades of octane, regular octane remains the product of choice for most consumers with an 82.2% market share in 2006. Midgrade is a mature product with a 2006 market share of 9.3%. Premium's market share ranks last at 8.4% in 2006. Midgrade's market share has exceeded that of premium in every year since 1995.[2]

What makes midgrade gasoline's pricing the object of our interest is that midgrade is a redundant product offering, easily and almost costlessly replicated by mixing existing regular and premium products. Indeed, this redundancy is widely known and exploited by: (i) appropriately mixing regular and premium feedstocks from the wholesale racks and combining them in a "wholesale rolling refinery" – an ordinary gasoline tanker truck; or (ii) appropriate just-in-time mixing at the retail pump from

[1] See "Premium Gasoline Overbuying in the U.S.: Consumer-Based Choice Analysis" 1993, Institute of Transportation Studies Research Report, University of California, Davis (available at www.uctc.net/papers/457.pdf)

[2] Energy Information Administration, "Petroleum Navigator" updated on 8/27/2007, http://tonto.eia.doe.gov/dnav/pet/pet_cons_refmg_d_nus_VTC_mgalpd_m.htm

separate underground regular and premium storage tanks.[3] Among producers and distributors, it appears to be widely understood that mixing octanes in the range of those eventually sold at retail is essentially a low-cost linear mixing technology.

In contrast to the widespread mixing of regular and premium feedstocks to form midgrade prior to retail purchase, seldom do we observe a consumer setting up to mix his own midgrade in his car's gas tank – a post-delivery "retail rolling refinery." That is, it is rare to see a consumer create a midgrade by buying from two retail feedstocks at a single retail gas station. This is true despite the overwhelming evidence that consumer midgrade mixing is almost uniformly the least costly way to buy retail midgrade. As we document, the aggregate cost savings if U.S. consumers created their own midgrade by mixing retail regular and premium, rather than buying pre-mixed midgrade, historically would have been as high $128 million in a single year (1998).[4] While the persistence of consumer demand for relatively overpriced pre-mixed midgrade may be a curiosity, consumer demand and rationality are not the focus of our investigation. It would be a mistake to suggest that buying the pre-mixed midgrade – rather than mixing it – is necessarily irrational. Paying $128 million a year for the convenience of pre-mixed midgrade may be quite rational, given the additional time and payment complications associated with mixing one's own midgrade.[5] Nevertheless, it is a bit puzzling how retail competition among stations coexists with the observed prices when retailers buy

[3] See, for example, www.eia.doe.gov/oiaf/servicerpt/fuel/gasoline.html, where the Department of Energy report titled "Gas Type Proliferation and Price Volatility" comments that handling additional gasoline types was accommodated by "partially eliminating the need to ship and store midgrade gasoline through the addition of in-line blending at terminals. Premium grade and regular grades were blended in appropriate ratios to create midgrade product as the material was loaded into trucks, and in some cases at the retail pump."

[4] For 1998, $128 million is almost 5% of total midgrade retail expenditures, and almost 1% of total retail gasoline expenditures (all octanes).

[5] One oil executive at a major U.S. petroleum company stated (in a private communication) that his personal belief was that customer convenience was a major reason midgrade was more expensive than the linear mix of the regular and premium. He also indicated that oil companies were aware of "rolling refinery" customers and still have not priced pre-mixed midgrade at the same price as "mix your own." In a separate conversation with an employee of a different energy company, it was suggested to us that pricing midgrade too close to regular can adversely affect the demand for premium. In related theoretical analysis, Tversky and Simonson (1993) find that introducing a third choice can influence the preferences between the other two choices (a violation of the Independence Axiom). The related argument for gasoline would be that introducing midgrade at a high price relative to regular makes premium more appealing. Whatever the reason for consumer demand for midgrade, our concern is the relationship between competition and the equilibrium price of the midgrade.

midgrade at a price equal to that paid for the corresponding mix of wholesale regular and premium, while they sell midgrade at a price higher than their customers would pay for the corresponding mix of retail regular and premium.

When we observe stations on four corners of an intersection, all posting triplets of prices for three octane grades with each triplet's displaying a relatively overpriced midgrade, we wonder how those retailers compete on price and still exploit consumers' willingness to pay for the convenience of pre-mixed midgrade. Why isn't midgrade's additional relative margin competed away in this market with instantaneous public price adjustments? Why isn't the convenience of a pre-mixed midgrade just an additional piece of consumer surplus created when the retailers compete on midgrade price? Why is pricing above the equivalent mixture price primarily a retail market phenomenon even though mixing costs are low in both the wholesale and retail markets? Even if gasoline retailers satisfy a zero profit condition overall, what notion of simultaneous price-quoting competition on the three grades results in midgrade's persistent subsidy to regular and/or premium? Why is the redundant product (midgrade) the chosen subsidizer even though midgrade is the only product of the three whose cross-subsidy to the other grades is readily apparent? Why is linear (arbitrage-free) pricing a widely-accepted normative quality of financial markets but not of retail gasoline markets? We believe these are interesting questions to consider when assessing how competitive the retail gasoline market actually is and, by analogy, how competitive other non-financial markets with post-purchase linear mixing and replication are. As in the analysis of a financial product market, our goal is to derive equilibrium relative price relationships when market participants have access to a low-cost post-production linear mixing technology. As is common in other investigations of potential post-purchase arbitrage trading, our approach does not directly address whether regular and premium are appropriately priced. We only consider whether an octane price triplet is consistent with equilibrium in a market with low mixing and trading costs.

The retail gasoline market has been, and continues to be, the subject of a great deal of scrutiny, including major state, federal and US legislative investigations. Particularly during times of dramatic price increases, gasoline distributors and retailers have endured almost continuous examination and monitoring.[6] Yet many investigations conclude that this near-commodity industry is competitive at the wholesale and retail levels.[7] Although distributor profits' reaching all-time highs in recent years has invited even more scrutiny, retailers are often the object of compassion; retailers and industry onlookers alike claim that retailers break even on gas sales and make most of their profits on snacks and other impulse purchases.[8]

The existing literature does not directly address our interest in pre-mixed midgrade's overpricing relative to post-purchase mixed midgrade. There is, however, previous research addressing other interesting aspects of retail gasoline pricing. Most closely related to our investigation in spirit, Borenstein (1991) documents different retail margins for leaded and unleaded gasoline. He offers three possible explanations: (i) cost-based differences possibly linked to the product's cashier time versus average bill; (ii) price discrimination varying by where the products are sold: full-service versus self-service stations; and (iii) systematic differences in customers' use of credit cards. Borenstein's 1991 work addresses a period mostly prior to the introduction of midgrade.[9] Shepard (1993) considers agency issues in gasoline stations and notes that company-owned stations tend to have lower retail prices than franchisees ("open dealers"). This is particularly true for premium octane where there is a statistically significant difference. Hastings (2004) looks at how competition changes when there is an acquisition that increases brand concentration. More recently, Barron, Taylor, and Umbeck (2000) consider the now-prevalent three octane configuration and prices from a qualitative perspective. They take octane as an ordinal measure of quality and seek to explain how consumer income and competition result in the observed prices. They find that premium markups are higher (than for regular) and that markups are positively related to income

[6] As an example, on April 17, 2006 New York Senator Charles Schumer called for an investigation to see whether oil companies and refiners were deliberately withholding gasoline production to sustain higher prices. The call for an investigation was given as oil prices reached new record highs. See, e.g. http://money.cnn.com/2006/04/18/news/economy/gas_price_investigation/index.htm.
[7] See, for example, the 2002 report by the U.S. Senate's Majority Staff on gasoline pricing.
[8] See, for example, the report by Wisconsin Radio Network at http://www.wrn.com/gestalt/go.cfm?objectid=442EC814-66A1-4408-9579B4A3EACD3963 or the commentary on North Carolina retailers' struggles at http://www.allbusiness.com/retail-trade/food-stores/4492041-1.html.
[9] The Borenstein (1991) study considers data from 1981 to 1989.

and the distance to competing stations.[10] In contrast to their ordinal use of octane grades, we consider the cardinal relationships between the octanes and prices of the three products, the redundancy of midgrade, and a notion of arbitrage-related restrictions for competitive equilibrium in markets with product replication, redundancy and negligible repackaging and mixing costs.[11]

Section 2 considers a hypothetical perfected consumer gasoline market allowing entry and trading at posted prices. We treat the wholesale and retail segments of the consumer gasoline market as separate markets and consider only trading strategies confined to a single segment, wholesale or retail. In such markets, the ability to costlessly mix offered octanes and resell (or value) the mixture at a posted price leads to the equilibrium restriction that prices be positively monotonic and convex in octane. Section 3 examines Department of Energy aggregate annual statistics and documents that both wholesale and retail prices exhibit positive monotonicity in octane. More interestingly, the DOE wholesale data suggest that prices are sufficiently close to arbitrage-free that arbitrageurs typically could not profitably exploit nonconvexities in wholesale prices. In contrast, DOE retail statistics suggest significant durable violations of arbitrage-free retail pricing. Section 4 considers station-level data purchased from the Oil Price Information Service (OPIS) and documents that the pricing behaviors suggested by the aggregate DOE data derive from station-level phenomena and are not a misleading byproduct of aggregation. Section 5 concludes and suggests several avenues for future research.

2. The Case for Positive Monotonicity and Convexity in Gasoline Prices

To understand retail midgrade gasoline pricing, it helps to consider what happens when one mixes octanes in the range of those sold to retail customers. If regular gasoline is

[10] Other related investigations include those of: (i) Marvel (1976) who considers the relationship between search costs and the cross-sectional dispersion of retail gasoline prices; (ii) Marvel (1978) who uses BLS data on 22 cities to document that markets with greater brand concentration have higher prices; (iii) Castanias and Johnson (1993) who analyze seven years of prices wars in the Los Angeles gasoline market; (iv) Borenstein and Shepard (1996) who document gas pricing dynamics; and (v) Borenstein, Cameron, and Gilbert (1997) who analyze asymmetries in retail price responses to oil price changes: retail prices respond faster when crude oil prices rise than when they fall.

[11] The absence of arbitrage as a necessary condition for equilibrium in frictionless financial markets is well-documented.

mixed with an equal amount of premium, what is resulting mixture's octane? As we've already suggested, evidence that one can mix premium and regular to produce midgrade can be found in a variety of places including regulators' recommendations that gas stations install only two underground tanks to avoid the "storage tank proliferation" associated with storing a third (or more) additional pre-mixed grades and reformulations.[12] Interestingly, many Sunoco station pumps have a dial that allows the consumer to select various (more than three) octanes that are mixed at the pump while being delivered into the consumer's gas tank.[13] For our purposes, however, the important question is not whether octanes can be mixed, but whether retail octane mixing is a *linear* technology.

More formally, define an "α mixture" as a gasoline mixture that has a mixing fraction of α in premium and 1-α in regular. Given that the regular, midgrade and premium octanes are denoted R, M and P, retail octane mixing is a linear technology when:

$$M = \alpha P + (1-\alpha)R = R + \alpha(P-R).$$

For typical retail offerings P-R = 6 (as is the case for R=87 and P=93).

Petrochemical industry research directly addresses the potential for only very mild nonlinearities in mixing a wide range of octanes at refineries.[14] After the refinery, at the wholesale and retail level, basically everyone uses linearity to calculate the octane of a blend of retail station octanes. Perhaps the most telling "evidence" that mixing retail grade octanes is essentially a linear technology comes from reported professional practice at the distributor level. In response to an inquiry, one tanker trucker reported that "The

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[12] See the Department of Energy report titled "Gas Type Proliferation and Price Volatility" (www.eia.doe.gov/oiaf/servicerpt/fuel/gasoline.html). A somewhat unrelated paper Manne (1952) considers how octane mixing relates to the Independence Axiom.

[13] For a colorful commentary on Sunoco's extensive offering of octanes by mixing at the pump, see (http://www.cartalk.com/content/columns/Archive/1994/June/07.html) taken from the National Public Radio Program "Car Talk."

[14] The two best references we've found incorporating non-linearities in octane prediction are given by Twu and Coon (2005) (updated 08/22/05 at http://www.simsci-esscor.com/NR/rdonlyres/78B44308-B734-46FF-B630-5B7587855651/0/Ron1.pdf with an improvement to a previously published non-linear octane predictor) and by S. Treiber, R. S. McLeod, Y. Faitakis and R. L. Hutchings at http://www.golden-lake.com/rfg_blnd.html (for a general discussion of the advantages of "at refinery" blending incorporating the non-linearities [interactions] due to the characteristics of the gas feedstock). The Twu and Coon paper provides an example showing how the non-linearity may change an intermediate octane from 92.025 to 92.02. There are known non-linearities in certain additives, in particular lead. An unrelated discussion appears in Manne (1952).

lower octane premiums are simply cut a bit with regular unleaded. That's also where "midgrade" or "plus" comes from; it's mixed as it goes on the tanker truck—35 percent premium and 65 percent regular in most cases."[15] Near linearity in mixing also can be conjectured and confirmed by consumers who experiment with mixtures for a car that "knocks" when regular octane is used.[16] Octane mixing linearity has even been discussed in mass media and on radio programs like "Car Talk".[17]

At the typical retail gas station, a midgrade consumer faces two perfectly substitutable versions of that retail station's midgrade: (i) the pre-mixed midgrade connected to the button or hose associated with the midgrade octane; and (ii) the perfect substitute midgrade mixed in the consumer's tank after its components are pumped into the tank using the buttons or hoses associated with the premium and regular octanes. Since both versions of the midgrade are from the same brand/station, even the detergents (in almost all cases) will not interfere with the perfect substitutability of the two types of midgrade (pre-mixed and mix-your-own).

Due primarily to the effect of altitude, octane levels for each grade can vary slightly by brand and by location. There are two pervasive octane menus for stations

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[15] http://theoildrop.server101.com/ubb/ultimatebb.php?ubb=get_topic;f=5;t=002231; Posted by fuel tanker man (Member # 4702) on August 25, 2004.

[16] When a gasoline product's octane level is too low for a given engine, one can hear "knocking." The knocking sound comes from the gasoline's igniting prematurely (before the sparkplug sparks) due to the compression being applied in the process. Octane is a measure of a gasoline product's tendency to ignite prematurely. Each car sold in the United States comes with a recommended minimum octane and a warning to fill the tanks only with complying octane levels. Avoiding knocking is important for engine wear and for fuel economy. However, besides avoiding knocking, higher octane confers no other advantage. Although it appears to be disputed by the American Petroleum Institute ("API: U.S. motorists not overbuying octane" *Oil and Gas*, Volume: 94, Issue: 39, Sep 23, 1996), there is relatively widespread belief that Americans have historically "overbought" octane. As we document below, in recent years there has been some attenuation in the demand for higher octanes. Our investigation of midgrade pricing takes as given whatever reason (valid or otherwise) the consumer has for demanding midgrade octane levels.

[17] Octane mixing and pricing has been discussed multiple times by "Car Talk" hosts Ray and Tom Magliozzi, in print and on the radio. In response to a question regarding alleged mixing behavior by "notorious cheapskate Andy Rooney," Tom Magliozzi stated that "most gas stations have only two underground gasoline tanks: one for the lowest octane, and one for the highest octane. Then the pump blends those two to produce everything in between." (*Philadelphia Weekly*, February 23, 2005, http://www.philadelphiaweekly.com/articles/9010). In a radio response, Tom asserted linearity in mixing in the retail range ("about 85 to 95") and Ray suggested that prices "ought to be linear, too, so you don't have to waste your time mixing gasolines." Ray then went even further to suggest that consumers go to their local gas station and ask an attendant to pump "one gallon of 87, and then one gallon of 93 --and ask him to keep alternating—until your tank is full. And when he complains, explain why you're doing it." (from "Click and Clack Talk Cars: Who Knew You Can Mix Octanes?, http://www.cartalk.com/content/columns/Archive/1994/June/07.html)

offering three octanes. For lower altitudes the octane triplet is usually 87, 89 and 93. For high altitudes (where knocking is less of a problem), the usual octane triplet is 85, 87 and 91. In both typical cases an $\alpha = 1/3$ mix would produce the midgrade. That is, 89 = 2/3(87) + 1/3(93) and 87 = 2/3(85) + 1/3(91). We can see that the tanker trucker's mixture mentioned ($\alpha = 35\%$) is a reasonable tanker mixing instruction. If all the octanes were perfectly accurate, an $\alpha = 35\%$ mix would produce 89.1 (low altitude) and 87.1 (high altitude). Given that about two-thirds of the gas is the lower regular octane, instructing a trucker to target a premium component in the mix of 35% (instead of 1/3) allows for a reasonable buffer for the "wholesale rolling refinery." The empirical results we present are robust to whether the midgrade is actually an $\alpha = 35\%$ or an $\alpha = 1/3$ mix.

While it is not directly relevant to our investigation of midgrade pricing, an additional advantage of consumers' mixing to produce intermediate octanes is that all specific intermediate octanes can be purchased. By choosing the appropriate mixing weights, the consumer can produce 88 if that is what is needed to avoid knocking. Mixing specifically for 88 will almost always be cheaper than buying pre-mixed 89 or mixing to produce 89. In fact, this is how we first became involved in experimenting with mixing octanes. One of the authors owned a car that knocked when using regular or midgrade at low altitudes. That car didn't knock when using premium octane. No in-depth knowledge of octane mixing was necessary to discover relatively quickly a least expensive knock-free mixture of the three offered octanes. In this type of casual investigation, it is almost immediately obvious that midgrade receives a zero weight in all least-cost knock-free mixtures.

With nearly exact linearity in the post-refinery mixing process for octanes available at retail stations, it is reasonable to ask whether pricing in this competitive "commodity" industry should also be linear in octane. Linearity in mixing octanes allows wholesalers and consumers to produce an arbitrary octane in the interval [R, P] simply by mixing the existing supply of regular and premium octanes at the same purchase point. The financial-markets-inspired abstraction we wish to examine is a hypothetical consumer gasoline market where the wholesale and retail segments are "perfected" in the sense of allowing market participants to enter and costlessly mix existing product offerings – that is, the perfected market allows participants to assemble portfolios of octane offerings and trade them without transactions costs and frictions.

In the usual analysis of perfected financial markets, a linear portfolio production technology allows financial market participants to generate a linear space of traded portfolios. The absence of arbitrage opportunities then dictates a linear price functional for those traded portfolios. In the typical financial markets setup, the technical details are simplified by allowing portfolios to combine arbitrary long and short positions (thus the linear space of traded portfolios). Given that a necessary condition for equilibrium is that the price functional not admit arbitrage trading opportunities, a linear price functional becomes a necessary condition for equilibrium. In complete markets, that linear price functional is unique. In summary, the availability of a post-purchase costless linear mixing technology leads to restrictions on equilibrium prices (linearity) directly related to precluding arbitrage opportunities.

For our hypothetical perfected gasoline market, we identify a product by its octane and physical location. That is, a gallon of midgrade at the Conoco on the southeast corner of Broadway and Table Mesa in Boulder, Colorado is a product. The Conoco (or any other) midgrade somewhere else in Boulder is a different product. This eliminates additional pricing restrictions arising from the ability to transport a mixed (or unmixed) product elsewhere. We will focus on the pricing restrictions implied by the ability to mix and trade octanes at (or for) a fixed geographic location, be it wholesale or retail.

Unlike many financial markets analyses, our hypothetical perfected gasoline market is not complete. Only octanes in the open interval (R,P) can be replicated by mixing components from a given station's offered octane triplet; there is no replication for that station's basis octanes R and P. Our hypothetical perfected gasoline trading market takes posted trading prices of R and P as given. As we will demonstrate, the absence of arbitrage leads to prices that are monotonically increasing in octane. We will assume that the price of R is less than or equal to the price of P and derive restrictions on the price of M given the prices of R and P. Our perfected gasoline market involves important short-selling restrictions inherited from the underlying octane mixing technology. Since it is not possible to mix negative quantities of gasoline, α must be between zero and one. For example, we cannot have a mixture with $\alpha = -.1$ of premium an $1-\alpha = 1.1$ of regular even though the mixing formula $(-.10)(93) + (1.1)(87) = 86.4$ suggests that this mixture of octanes would yield an 86.4 octane midgrade.[18] This is analogous to a restriction on short selling in financial markets. So for our perfected gasoline market, we will require that equilibria preclude only those feasible arbitrage trading opportunities that involve mixing fractions in the closed interval [0,1]. The consequence of these restrictions on portfolio weights permitted for trading strategies is that the equilibrium relative pricing restrictions will be strictly weaker than those established for financial asset portfolios traded in perfected financial markets. As a consequence, for our perfected gasoline trading market, it is not necessary that equilibrium prices be linear (in octane) even with free entry and costless mixing and trading. While weaker than linearity, we can still characterize relative pricing constraints for each target octane in (R,P) as a function of the prices of R and P or as a function of any other octanes in [R, P] from which that target octane can be mixed.

Consistent with the industry's product labeling, we assume that any higher octane can be legally sold as a lower octane grade since the octane ratings are minimums. For our perfected retail gasoline market, we ask what relative price restrictions preclude a hypothetical arbitrageur's (or entrant's) ability to enter the consumer gasoline market, replicate wholesale or retail offered octanes by mixing, and profit by buying and selling at equilibrium posted prices.

We are not denying that arbitrage-oriented retail intermediaries would face entry costs or even small marginal mixing costs; nor are we asserting that a retail station owner would allow someone to stand on her property, buy her regular and premium, and simultaneously sell an "after-market-mixed" midgrade at lower than her posted midgrade price. However, if the reality of such frictions obviates the consideration of any price constraints necessary to preclude arbitrage, there are many important theoretical asset pricing models that should be similarly ignored. For example, the Black and Scholes option pricing model is derived assuming there are no arbitrage opportunities, even when

[18] Additionally, we do not have a low-cost octane skimming technology analogous to the butter fat skimming technology referenced in Modigliani and Miller. Re-refining a given octane to separate it into components having higher and lower octanes than the original would be prohibitively expensive for a retail arbitrageur. All of these restrictions on mixing and skimming technologies are constraints on the strategies that arbitrageurs can reasonably undertake in trading and mixing post-production gasoline products.

the hypothesized arbitrage opportunity would involve infinite transactions costs in the real world (e.g., continuous time delta hedging).[19] We denote the nonnegative equilibrium vector of posted prices by π, with subscripts to indicate the octane grade.

Lemma (Weakly Positive Monotonicity): Equilibrium posted prices in a perfected gasoline trading market are a nondecreasing function of octane.

> Proof: Consider two octanes A and B such that A > B. Suppose in order to produce a contradiction that $\pi_A < \pi_B$. Consider the following transaction: buy A at the posted price of π_A and simultaneously sell it for the posted price of π_B (a perfectly legal option since A has a minimum octane of B). The riskless (arbitrage) profit on such a set of trades is $\pi_B - \pi_A > 0$. The absence of such trading opportunities is a necessary condition for equilibrium. This contradiction establishes $\pi_A \geq \pi_B$.

Theorem 1 (Triplet Prices): For a perfected consumer gasoline market restricted to offering only three octanes (P > M > R), if α_M is the α-mix of premium and regular exactly yielding the midgrade octane $M \in (R, P)$, equilibrium prices exhibit weakly positive monotonicity ($\pi_P \geq \pi_M \geq \pi_R$) and a quoted midgrade price weakly below the α_M mix price so that $\pi_M \in [\pi_R, \alpha_M\pi_P + (1\text{-}\alpha_M)\pi_R]$.

> Proof: $\pi_P \geq \pi_M \geq \pi_R$ follows from the application of the Lemma by applying (A=M, B=R) and then (A=P, B=M). Given the offering of a single intermediate octane, the only potential arbitrage trade not covered by the Lemma involves selling midgrade mix to compete with the station's pre-mixed midgrade. The posted midgrade price is π_M and the retail product can be replicated using an α_M-mix. If $\pi_M > \alpha_M\pi_P + (1\text{-}\alpha_M)\pi_R$, the arbitrageur makes a strictly positive profit by buying the α_M-mix at posted retail prices and selling it at the posted midgrade price $\pi_{M.}$ Precluding profitable arbitrage dictates that $\pi_M \in [0, \alpha_M\pi_P + (1\text{-}\alpha_M)\pi_R]$ We have already established that $\pi_M \geq \pi_R$. Consequently, equilibrium (with the necessary absence of arbitrage opportunities) requires $\pi_M \in [\pi_R, \min(\alpha_M\pi_P + (1\text{-}\alpha_M)\pi_R, \pi_P)] = [\pi_R, \alpha_M\pi_P + (1\text{-}\alpha_M)\pi_R]$ and $\pi_P \geq \pi_M \geq \pi_R$. ■

Theorem 2 (Continuum of Prices): For a perfected consumer gasoline market, if market participants (sellers and potential arbitrageurs) quote prices for the entire convex set of feasible octane mixtures $O \in [R,P]$, then posted equilibrium prices must be weakly increasing and weakly convex in octane.

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[19] If these caveats regarding the costs of actually arbitraging financial markets fail to provide an impetus for our related arbitrage inquiry into the gasoline markets, one could consider our investigation to be about why the Law of One Price is violated in the retail midgrade pricing even though consumers have zero search costs to find two differently priced supplies of the exact same midgrade. For a discussion search-cost-based violations of the Law of One Price, see Burdett and Judd (1983).

Proof: That prices are weakly increasing in octane follows directly from the Lemma. To prove convexity, suppose (in order to create a contradiction) that there exists $\alpha \in [0,1]$, $A \in [R,P]$ and $B \in [R,P]$ such that equilibrium prices satisfy $\pi_{\alpha A+(1-\alpha)B} > \alpha\pi_A + (1-\alpha)\pi_B$. Consider the following transaction: buy components A and B in the proportions α and $(1-\alpha)$ and combine them to produce a mixture having octane $\alpha A+(1-\alpha)B$ while simultaneously agreeing to sell the resulting mixture for $\pi_{\alpha A+(1-\alpha)B}$. The riskless profit thereby obtained is $\pi_{\alpha A+(1-\alpha)B} - (\alpha\pi_A + (1-\alpha)\pi_B) > 0$ per gallon of the mixture. This arbitrage opportunity conflicts with the absence of arbitrage opportunities in equilibrium. Therefore it must be that $\pi_{\alpha A+(1-\alpha)B} \le \alpha\pi_A + (1-\alpha)\pi_B$, which establishes convexity for prices on the closed interval [R,P].

In the usual financial markets case, the ability to take arbitrary long and short positions in financial assets establishes the convexity of Theorem 2 but also establishes concavity (by inverting the buy and sell positions). This leads to the well-known linear relationship between portfolio prices and payoffs in arbitrage-free markets. Here we cannot employ negative weights to octane offerings (because we can't produce regular from premium and midgrade), so arbitrage arguments are less powerful in restricting the functional form of equilibrium arbitrage-free prices. Linear-in-octane prices are arbitrage-free. However, since only convexity and monotonicity are required for equilibrium in our perfected consumer gasoline market, linear prices are not the only ones that preclude arbitrage. We believe there is a strong normative case for suggesting that the underlying low-cost post-purchase linear mixing technology (like the assumed low cost linear portfolio formation technology in financial asset markets with low replication costs) invites a meaningful consideration of arbitrage restrictions on the posted retail gasoline prices.

Our analysis suggests that equilibrium midgrade prices in a perfected wholesale or retail market that allows arbitrage trading would exhibit prices that are positively monotonic and convex in octane. Formally, for both wholesale and retail prices, we wish to examine the null hypotheses:

H1: Posted gasoline prices exhibit positive monotonicity in octane.

H2: Posted gasoline prices are drawn from a convex function $\pi:[R,P]\rightarrow R^+$.

The remainder of this paper examines the evidence regarding H1 and H2 in Department of Energy annual data and OPIS daily data.

3. DOE-EIA Aggregate Annual Data

We begin our empirical investigations by considering Department of Energy (DOE) data. The DOE data we use were obtained from the DOE's Energy Information Administration (EIA). The EIA provides the "official energy statistics from the U.S. Government." We pulled the annual data for 1996 to 2006 using EIA's update dated August 27, 2007. Table 1 presents basic annual volume and price statistics for the DOE/EIA data.

Table 1. DOE/EIA Descriptive Statistics

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Retail Sales (Daily Gallons 000s)											
Regular	37,153	40,533	41,712	41,595	43,637	45,036	46,228	47,333	44,506	46,360	48,160
Midgrade	9,545	9,692	10,115	9,901	8,576	8,268	8,341	7,762	6,431	5,945	5,472
Premium	8,413	8,426	9,614	8,677	6,911	7,096	7,521	7,124	5,915	5,242	4,933
Average Wholesale Prices (cents)											
Regular	67.3	65.9	48.4	60.5	92.8	84.8	80.2	96.6	126.1	165.5	194.7
Midgrade	71.1	69.8	52.6	64.0	96.2	89.0	84.0	100.6	129.9	168.9	199.0
Premium	76.2	75.0	57.4	68.6	101.7	93.6	89.9	106.5	136.6	176.4	209.8
Average Retail Prices (cents)											
Regular	81.2	80.0	62.5	73.0	106.6	99.6	91.6	111.1	140.1	180.0	210.0
Midgrade	89.0	88.5	71.6	81.6	115.0	109.0	100.5	120.1	148.6	188.0	218.3
Premium	97.0	96.4	79.4	88.8	122.7	116.5	108.9	129.1	158.8	199.6	230.9

An example of our calculations on the average prices helps to clarify our approach. For 2002, the wholesale price triplet (corresponding to R, M, P) is (80.2, 84.0, 89.9). We also observe a retail price triplet of (91.6, 100.5, 108.9). These average prices are consistent with positive monotonicity in octane (H1). We will have to do a few calculations to see if they are consistent with convexity in octane.

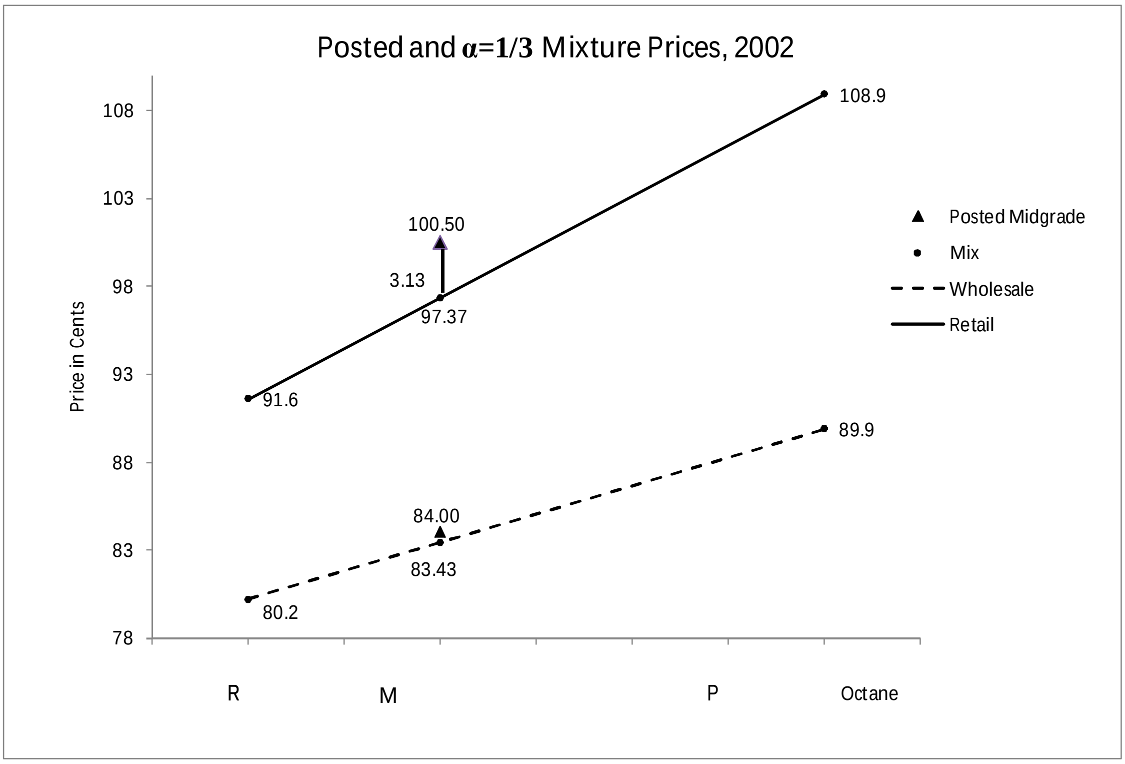
The $\alpha=1/3$ wholesale midgrade mix price is 83.43 =(1/3)(89.9)+(2/3)(80.2). The average posted price of 84.0 is .57 cents higher than the mix price. This is evidence against convexity in the wholesale price if the appropriate mix is $\alpha=1/3$. Although these are nationwide averages, the concept is that in a perfected wholesale trading market, buying the $\alpha=1/3$ wholesale mix at 83.43 cents per gallon and selling it for the posted wholesale midgrade price of 84.00 would have resulted in a riskless profit of .57 cents per gallon. For the 2002 retail triplet, the midgrade mix price is 97.37. Buying the retail α=1/3 mix at 97.37 and selling it for the posted midgrade price of 100.50 would have resulted in riskless profit of 3.13 cents per gallon.

To calibrate the economic relevance of the midgrade arbitrage to which we refer, consider a midgrade buyer making a 17-gallon purchase. With a 3.13 cents per gallon spread between the posted midgrade price and the cost of an equivalent mixture of premium and regular, the 17-gallon consumer saves 53.21 cents per fill-up. Assuming that scanning the same credit card a second time and changing hoses takes an additional 30 seconds, the savings would accrue at $63.85 per hour of mixing time. As this is an after tax savings, the before tax implied wage for a mixing buyer in a 40% combined marginal tax bracket would be $106.42 per hour risk free. While this savings reflects the nationwide average, it doesn't necessarily indicate how lucrative mixing can be. A recent observation in the Los Angeles metropolitan area indicated a 20-cent spread from regular to midgrade and a 10-cent spread from midgrade to premium. The effective before-tax wage rate for a 17-gallon buyer's mixing at this station is $340 per hour risk free.[20] Clearly, the potential for arbitrage exceeds what one would consider to be reasonable transactions costs for conducting the arbitrage.[21] Figure 1 provides a graphical representation of the nonconvexity implied by the triplets of average wholesale and retail prices.

[20] For an alternative benchmark on how much the 3.13 cents per gallon might matter, consider that there are many consumers who spend more than 30 seconds searching for alternative stations with lower prices. The certain savings providing by the 30 second midgrade arbitrage (averaging 3.13 cents per gallon) is easier and may provide greater average savings than searching for alternative stations with lower prices.

[21] For those who wish to see such a transaction cast in a more traditional arbitrage framework, for any given buyer, we can think of that buyer having two subsidiaries: (i) the ultimate consumer subsidiary; and (ii) the midgrade arbitrage subsidiary. The midgrade arbitrage subsidiary is instantaneously and costlessly recreated each time the ultimate consumer subsidiary wishes to purchase midgrade. The midgrade arbitrage subsidiary enters into an agreement to sell midgrade at the posted price to the ultimate consumer subsidiary. While simultaneously buying regular and premium from the retail station at posted prices, the midgrade arbitrage subsidiary sells the midgrade mixture to the ultimate consumer subsidiary and books a riskless pre-tax profit amounting to $106.42 (or $340) per hour while conducting arbitrage. The riskless profit the midgrade arbitrage subsidiary makes in buying from the retail station and selling to the ultimate consumer subsidiary is exactly the type of arbitrage transaction precluded in the usual consideration of the pricing restrictions suggested by arbitrage trading.

Figure 1. Nonconvexity of Posted Midgrade Prices (Wholesale)

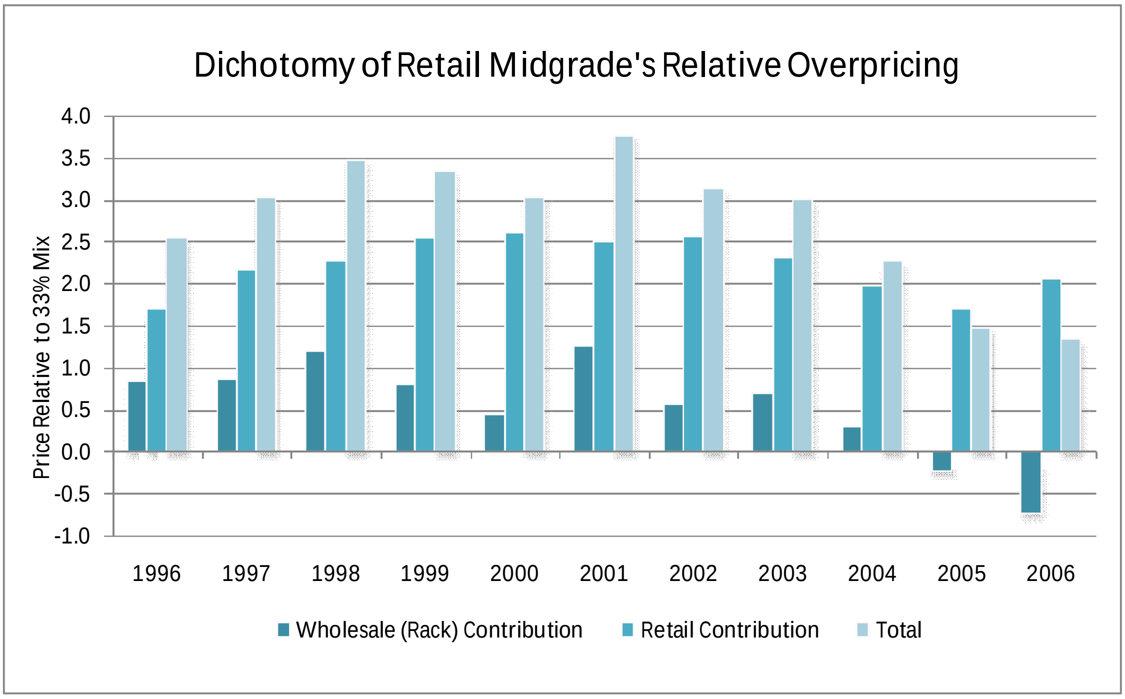


Source: Energy Information Administration, 8/27/07

Since we have reason to believe that some distributors may instruct tanker truckers to use an α=35% mix for the midgrade, we also examine convexity using this richer mix. At α=35%, the wholesale mix price is 83.60 =.35*89.9+.65*80.2. The average posted wholesale midgrade price of 84.0 remains .41 cents higher than the cost of this α=35% mix price. The retail mix price for α=35% is 97.6 (=.35*108.9+.65*91.6). It remains 2.85 cents below the average posted retail midgrade price of 100.50. Figure 1 would not be substantially different for an α=35% mix.

If we consider the gap between posted midgrade price and the mix price as "midgrade's relative overpricing" we can decompose the mispricing into its wholesale and retail components. For 2002 and α=1/3, we have a final relative mispricing of 3.13 cents per gallon that comes from an inherited wholesale mispricing of .57 cents and 2.56 cents of supplemental retail mispricing. Figure 2 presents this decomposition for each of the years in our DOE/EIA sample assuming that α=1/3. The first and darkest bar in a group of three is the wholesale contribution (e.g., .57 cents in 2002). The middle bar is the retail contribution (e.g. 2.56 cents in 2002). The final and lightest bar in the threesome is the total relative overpricing paid by the consumer (e.g. 3.13 cents in 2002). It is the sum of the previous two bars in the cluster of three. Note that the most common configuration is minor wholesale mispricing (nonconvexity) combined with more significant retail mispricing. Interestingly, in recent years wholesalers appear to have priced midgrade below its mixture costs (negative dark bars in 2005 and 2006) while retailers continued to post high midgrade prices (relative to the mix price). The retailer's contribution to the ultimate mispricing faced by consumers in 2006 is similar to previous years even though wholesalers actually priced below the mix (in averages).

Figure 2. Dichotomy of Midgrade Overpricing



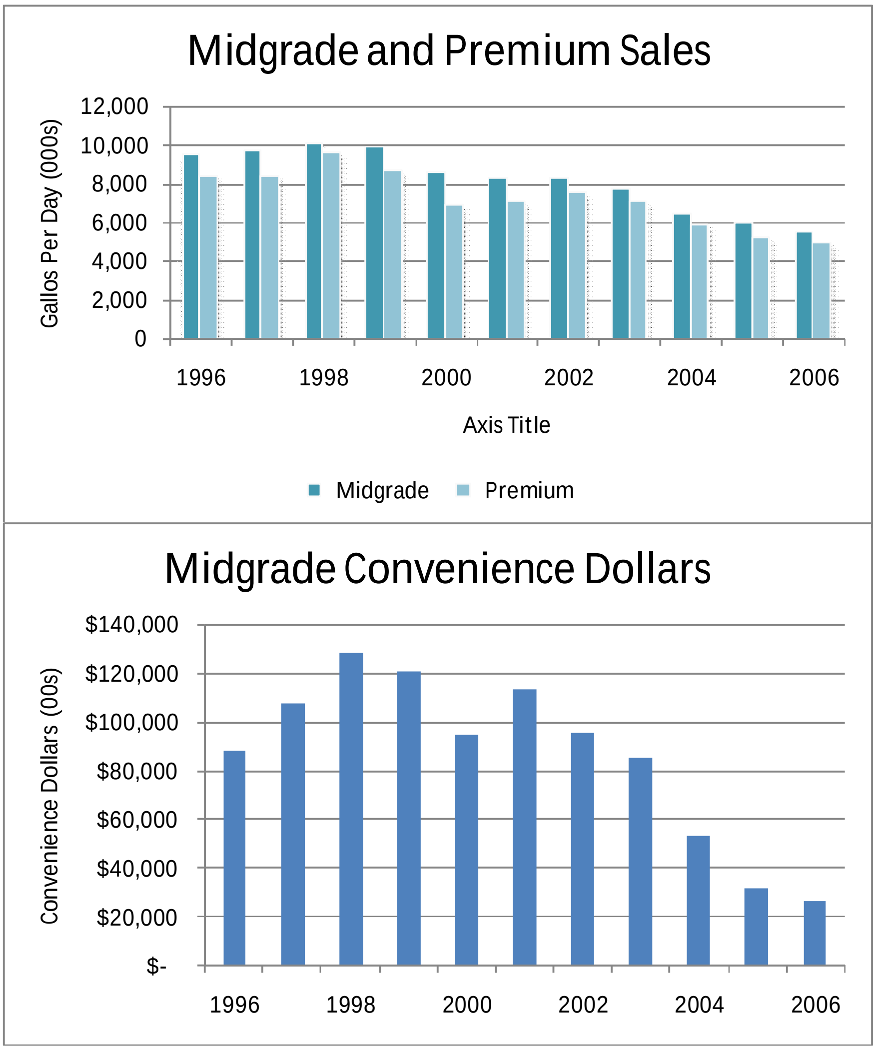
Source: Energy Information Administration, 8/27/07

Occasionally gas stations sell octane grades that may imply a mixture reference different from α =.35 or α=1/3. While this is nonstandard and rare, it could be a contributor to minor nonconvexities in average wholesale prices. However, average wholesale prices did satisfy convexity during the recent gas price increases of 2005 and 2006. Whatever the reason for average small over- or under-pricing at the wholesale level, midgrade's relative overpricing at the retail level is: (i) far too high to be explained by problems aggregating different octane regimes or distributor reference mixtures; and (ii) stands in stark contrast to wholesale pricing which is clearly very close to mixture (and arbitrage-free) pricing.

To look at the aggregate economic impact of retail midgrade's mispricing, we multiply the quantity consumed by the amount of "overpricing." For example, in 2002 DOE data indicate that 8,341,000 midgrade gallons per day were sold by U.S. retail outlets. The annual total is therefore a little over 3 billion gallons which on average are overpriced (relative to the mix) by 3.13 cents per gallon. The cost of the consumer convenience of dispensing pre-mixed midgrade (relative to mixing using the retail prices for regular and premium) is therefore about $95.4 million in 2002 alone. The cumulative convenience cost for pre-mixed retail midgrade reached about $1 billion in 2006. It is puzzling that consumers are willing to pay these convenience costs and perhaps even more puzzling that competition between retailers has not eliminated them.

Figure 3 displays the annual convenience costs that consumers paid aligned with aggregate midgrade retail volume. Not surprisingly, recent gas price increases have resulted in lower consumption of midgrade (and premium). Pre-mixed convenience costs have fallen, in part due to this decline in midgrade demand, and in part due to the recent wholesale under-pricing of midgrade (relative to a mix of wholesale prices) evident in Figure 2.

Figure 3. Higher Octane Sales and Midgrade Convenience Dollars



Source: Energy Information Administration, 8/27/07

4. OPIS Station-Level Daily Data

While the annual volume and average prices given by the Energy Information Administration suggest a systematic tendency for overpriced retail midgrade (relative to the price of an equivalent mix of retail premium and regular), it is instructive to consider specific station-level data where we can match a given station's retail prices to the wholesale prices it faces. To do so, we purchased station-level data from a commercial data provider specializing in the retail and wholesale fuels market, the Oil Price

Information Service (OPIS). According to its promotional material, OPIS is “the only provider of U.S. spot, rack and retail prices - giving us a complete picture of the marketplace that is rivaled by no other petroleum information supplier”. Appendix A gives a summary of OPIS’ antitrust, independence and data collection and integrity policies. All data (DOE-EIA and OPIS) referenced in this study were obtained from public data providers. Only the DOE-EIA data, however, were provided free of charge.

Our specially constructed OPIS data set spans 1996 to 2001 and contains 1,654,521 daily station-level retail price triplets with matched station-specific wholesale price triplets. During the period of overlap, OPIS means can serve as a double check on the means provided by the EIA. More importantly, the detail in the OPIS data permits us to examine other statistics such as medians and other quantiles and to construct statistics that represent how a typical station behaves in setting a price triplet. In this context, we can be much more certain that the interpretations we suggested for the annual EIA averages are appropriate and not the result of aggregation problems. Our disaggregated station-level OPIS data allow a direct examination of the retail midgrade overpricing chosen by a typical retailer and experienced by a typical consumer. They also permit an examination of station-level default pricing behaviors. For example, we can document retailers’ tendencies to quote triplets having a price increment for the *two* point octane increase from regular to midgrade that equals the price increment for the *four* point octane increase from midgrade to premium. We can also investigate brand-specific and regional-locational tendencies, although much of this we relegate to future research.

There are several ways to construct observations from the OPIS data. The data constitute a very unbalanced panel. Some stations’ prices only appear in the data set for a few days. Others have a long and almost continuous presence. We do not believe that the appearance (or lack thereof) of a station’s observations in a given period is related to survivorship issues. Many stations appear, disappear and then reappear in the data set. Rather, we believe that OPIS is collecting data they believe to relevant for their reports, and their procedures may involve regional sampling techniques that result in the rather sparse character of the panel. Appendix A provides some additional information on OPIS’s approach to collecting data from up to 120,000 stations targeted by their daily data collection.

An obvious first approach to constructing observations is to let each of the 1,654,521 station-day records in our OPIS dataset be an observation. However, due to the differences in how many observations a given station has, the resulting statistics would be heavily influenced by those stations having long time series in the data set. A station's time series would also exhibit a high level of autocorrelation in price behavior. While such an approach suggests some caution in interpretation, we will designate the "each station-day is an observation" as the Pooled Time-Series Cross-Section (PTSCS) statistics. As a second approach, we can compress a given station's entire time series into each variable's mean. Then, we can examine the cross-sectional distribution of station means to describe regularities in stations' average behaviors. In such an approach each of the 40,381 stations represented in our OPIS dataset gets an equal weight irrespective of how many days that station appears in the dataset. We refer to the statistics resulting from this approach as Station Cross-Section (SCS) statistics. Our final approach in constructing an observation for statistical tests is to take the station-level means used in the SCS approach and compute the mean of means for all stations within a brand. This results in only a single observation for each of the 138 retail brands in our OPIS dataset. We refer to the statistics taken from this approach as the Brand Cross-Section (BCS) statistics.

When initially examining the OPIS data we noticed that, with very few exceptions, U.S. retail prices are effectively on a one cent grid (ending with .9). This is not the case for wholesale prices, which can vary by one thousand of a cent or less. The difference in the fineness of prices at the wholesale and retail levels is worth remembering when we consider medians and modes.

Table 2. PTSCS Statistics (each station-day is an observation)

1,654,521 Observations

	10th %-tile	Median	90th %-tile	Mean	Mode
Retail Prices					
Regular	107.9	139.9	167.9	139.7	139.9
Midgrade	118.9	149.9	177.9	149.8	149.9
Premium	127.9	159.9	186.9	159.0	159.9
Midgrade - Regular	8.0	10.0	13.0	10.1	10.0
Premium - Midgrade	6.0	10.0	10.0	9.2	10.0
Wholesale Prices					
Regular	54.5	87.1	107.5	83.8	103.5
Midgrade	58.8	90.8	111.3	87.4	107.2
Premium	64.6	96.5	117.5	93.7	113.9
Midgrade - Regular	3.0	3.6	4.1	3.6	3.7
Premium - Midgrade	5.5	6.3	7.0	6.3	6.7
Mixture Savings					
Retail					
1/3 Mix Savings	2.3	3.3	6.0	3.7	3.3
35% Mix Savings	2.0	3.0	5.6	3.4	3.0
50% Mix Savings	-0.5	0.0	2.5	0.5	0.0
Rack					
1/3 Mix Savings	-0.1	0.2	0.8	0.3	0.2
35% Mix Savings	-0.3	0.1	0.6	0.1	0.1
50% Mix Savings	-1.8	-1.4	-0.8	-1.3	-1.5
Spread Statistics					
Equal Retail Spreads				54.5%	
Bigger Mid-Reg Spread				33.4%	
Bigger Pre-Mid Spread				12.1%	
10-10 Spreads				43.1%	

For the EIA annual aggregate data we documented annual wholesale midgrade overpricing relative to an α=1/3 mix ranges from -0.7 in 2006 to +1.3 in 2001. In the PTSCS statistics of Table 2, we see that the overall mean wholesale midgrade overpricing is .3 cents for an α=1/3 mix and .1 cents for an α=.35 mix. For wholesale prices, working with station-level data suggests that wholesale midgrade prices are very close to those implied by an appropriate mix of wholesale premium and regular prices. For retail prices, the EIA data suggest final retail midgrade overpricing of 1.3 in 2006 to 3.8 in 2001. Table 2 indicates that the OPIS sample suggests retail midgrade overpricing of 3.7 relative to an α=1/3 and 3.4 relative to an α=.35 mix. The OPIS data in the aggregate confirm the notion that the mispricing is primarily a retail phenomenon not inherited from the wholesale pricing structure.

Table 2 also provides other statistics on the distribution of prices including the 10th, 50th, and 90th percentiles and the mode. The relative overpricing of midgrade retail is a robust finding. In fact, we can see that the retail is usually priced as though it were an α=.50 mix of premium and regular. In the "Spread Statistics" section of Table 2, we compare the difference (spread) between midgrade and regular to that between premium and midgrade.[22] We find that retail octanes are priced as though retail midgrade is an α=.50 mix in over 54.5% of the 1,654,521 observations. In 79% (43.1/54.5) of those equal retail spread observations we find that the spread is 10 cents. Perhaps even more interesting, we also find that when the midgrade is not halfway between the regular and premium it is more common to have the midgrade-regular price spread larger than the premium-midgrade price spread. The troubling part is that the midgrade-regular octane spread (2) is typically half of the premium-midgrade octane spread (4). At the wholesale level, midgrade appears to be priced as though it is about an α=.35 mix. This confirms the notion that midgrade mispricing is predominantly a retail phenomenon.

In terms of our hypothesis H1, we observe prices that appear to be consistent with positive monotonicity in octane at both the wholesale and retail level. For our hypothesis H2, we provide evidence that wholesale prices, while occasionally consistent with

[22] While it makes sense to consider modes for wholesale variables, they are not as informative as they are for retail. For wholesale, prices are not restricted to an integer grid (ending in .9). Wholesale prices vary more continuously with many more digits, and are not obviously clustered (based on our cursory review of the raw data). Consequently, we do not report spread statistics for wholesale prices.

convexity in octane (negative mispricing at the 10th percentile), technically violate convexity most of the time. However, the violations of convexity in wholesale prices appear to be small in magnitude (positive mispricing of .1 cents on average). This suggests that arbitrage profits could easily be limited by slight imperfections and transactions costs in a hypothetical market for trading wholesale octanes. In contrast, the noncovexity in retail octane prices is pervasive, reaching 6 cents a gallon at the 90th percentile. This suggests that arbitrage profits might not be overcome by modest imperfections and transactions costs in a hypothetical market for trading retail octanes. In summary, given modest transactions costs, wholesale prices are roughly consistent with arbitrage-free pricing while retail prices appear to be ripe for exploitation in a perfected retail octane trading market. Why entrants do not arise to exploit midgrade's overpricing relative to an equivalent mix is a puzzle. How price competition adds this relative pricing distortion, which amounts to a subsidy from midgrade to the other octanes, is not clear (at least to us).

Table 3 presents the corresponding statistics when we reduce each station's time series to a single mean for the given variable and then characterize the cross-sectional distribution of those station means. Although there are slight differences, the results are largely the same. Our conclusions are robust to allowing each station to contribute only a single observation to the statistics. It is interesting that retail 10-10 spreads are still quite common (40.6%). This suggests that the behavior is specific to stations where it tends to be pervasive. If a station priced at 10-10 spreads most, but not all, of the time, then its station time-series average spreads would most likely be something other than 10-10.

Table 3. SCS Statistics (each station time series forms a single observation)

40,381 Observations

	10th %-tile	Median	90th %-tile	Mean	Mode
Retail Prices					
Regular	112.2	137.4	160.9	137.0	139.9
Midgrade	122.2	147.2	171.1	147.0	149.9
Premium	131.3	156.6	180.4	156.3	159.9
Midgrade - Regular	8.0	10.0	12.0	10.0	10.0
Premium - Midgrade	6.8	9.8	10.4	9.2	10.0
Wholesale Prices					
Regular	60.1	84.5	101.5	82.0	89.3
Midgrade	63.6	88.0	105.0	85.5	92.9
Premium	69.8	94.4	111.3	91.8	99.0
Midgrade - Regular	3.2	3.6	3.9	3.5	3.6
Premium - Midgrade	5.8	6.3	6.7	6.2	6.0
Mixture Savings					
Retail					
1/3 Mix Savings	2.3	3.3	5.3	3.6	3.3
35% Mix Savings	2.1	3.0	4.9	3.3	3.0
50% Mix Savings	-0.5	0.1	2.0	0.4	0.0
Rack					
1/3 Mix Savings	0.0	0.3	0.5	0.3	0.4
35% Mix Savings	-0.1	0.1	0.4	0.1	0.3
50% Mix Savings	-1.6	-1.4	-1.1	-1.3	-1.2
Spread Statistics					
Equal Retail Spreads				50.4%	
Bigger Mid-Reg Spread				33.4%	
Bigger Pre-Mid Spread				16.2%	
10-10 Spreads				40.6%	

Table 4 presents the statistics where we create a brand observation by taking the within-brand cross-sectional mean of individual station time series means for a given variable (a brand's cross-sectional mean of station time-series means). Our OPIS dataset contains 138 Brands.

Table 4. BCS Statistics: Each Brand is a Single Observation

138 Observations

	10th %-tile	Median	90th %-tile	Mean	Mode
Retail Prices					
Regular	117.3	139.4	154.2	138.1	NR
Midgrade	126.1	148.9	164.2	147.5	1.1
Premium	134.7	158.0	173.8	156.5	1.5
Midgrade - Regular	7.1	9.6	10.5	9.4	0.1
Premium - Midgrade	7.1	9.2	10.2	9.0	0.1
Wholesale Prices					
Regular	68.1	86.6	96.7	84.2	NR
Midgrade	71.6	90.4	100.4	87.8	NR
Premium	77.5	96.5	106.6	94.0	NR
Midgrade - Regular	3.3	3.6	3.8	3.6	NR
Premium - Midgrade	6.0	6.3	6.5	6.3	NR
Mixture Savings					
Retail					
1/3 Mix Savings	2.3	3.3	4.1	3.2	0.0
35% Mix Savings	2.0	3.0	3.8	2.9	0.0
50% Mix Savings	-0.5	0.1	0.9	0.2	0.0
Rack					
1/3 Mix Savings	0.1	0.3	0.4	0.3	NR
35% Mix Savings	0.0	0.1	0.3	0.1	NR
50% Mix Savings	-1.5	-1.4	-1.2	-1.3	NR
Spread Statistics					
Equal Retail Spreads				58.4%	
Bigger Mid-Reg Spread				23.9%	
Bigger Pre-Mid Spread				17.6%	
10-10 Spreads				41.2%	

At this level modes cease to have much meaning. Even though retail prices tend to be taken from a one cent grid, the averaging of averages almost completely hides the effect of the grid leaving almost continuous variation in the resulting variables used for a brand observation. The difficulty in using modes is apparent in comparing the 109.9 for midgrade and the 154.9 for premium. Such numbers are really not comparable. Note that we observe 41.2 % 10-10 spreads even after all the averaging.

Our conclusion that the aggregate and station-level data support positive monotonicity of prices in octane is robust to how we weight station-day observations. Similarly, the station-level data support our conclusion that wholesales prices, while technically violating the convexity of prices in octane (H2 for wholesale prices), do not appear to stray far from arbitrage-free pricing. In contrast, retail prices exhibit statistically and economically significant departures from arbitrage-free pricing due to their violation of convexity (H2 for retail prices).

As further evidence for the normative notion that observed *retail* gasoline prices should preclude arbitrage (as though octanes were traded in a perfected trading market), we present abundant evidence that *wholesale* prices do appear to be consistent with the perfected-market required positive monotonicity and convexity in octane. Only retail prices regularly, persistently, and almost uniformly violate convexity. Retail prices, while exhibiting positive monotonicity (precluding perfect-market arbitrage trading involving selling a higher octane labeled as a lower one), appear to be taken from a price function that is concave, rather than convex, in octane (thus admitting perfected-market arbitrage trading involving intermediate octanes). Retail prices are almost uniformly in a configuration that would give rise to incentives for intermediaries to enter and attempt to capture midgrade market share. More immediately, the prices suggest that, in the absence of mixing costs, almost all retail customers should mix their own midgrades rather than buy the stations' pre-mixed midgrades.

We are puzzled by how publicly posted price triplets incorporating retail midgrade prices roughly 3 to 4 cents a gallon above the mixing cost persist in a market allegedly engaged in price competition with low mixing costs. While convenience can explain why consumers are willing to *pay* above-linear midgrade prices rather than mix premium and regular, it does not explain why retailers are able to *charge* such prices in a competitive market with constant entry and exit. The arbitrage and entry incentives such prices introduce may not represent actual trading opportunities; they do suggest questions about the industrial organization giving rise to such prices, and whether that structure can survive in a market that truly engages in price competition and permits new entry. While retailers may somehow otherwise compete away their bottom line profits, why is it that midgrade is relatively so profitable? Bertrand competition on all three grades should result in prices close to marginal costs. The marginal costs faced by retailers are, by a very close approximation, the wholesale prices they pay for the marginal product. We've shown that wholesale prices appear to suggest that arbitrage buying and selling at posted prices, were it allowed, would not be sufficiently profitable to entice arbitrage traders to enter. In contrast for retail market participants, while we can accept that there may be stiff competition between retail stations, there seems to be provocative evidence that the competition is not equal across octanes.

5. Conclusion

Retail midgrade gasoline appears to be overpriced relative to the other two grades. How this is sustained in a near-commodity industry engaged in price competition is not clear. Consumers and new entrants can cheaply replicate midgrade from available premium and regular supplies. At the wholesale level there is little, if any, financial incentive to offer to mix post-production midgrade and sell it at the prevailing wholesale price for midgrade. In contrast, there appears to be ample incentive to do so at the retail level. The type of (price) competition that prevails at retail stations and durably supports the relatively overpriced midgrade is not the usual Bertrand price competition or a near neighbor. Simple variations on Bertrand competition do not appear to support this result. Cournot equilibrium is also not an obvious answer to the question of why the midgrade is relatively overpriced. Whatever model of competition one wishes to consider, it remains perplexing that prices so far from arbitrage-free are pervasive and persistent.

Our results provide some contrast to financial markets. We see that linear mixing leads to linear pricing in the wholesale market, but does not lead to linear pricing in the retail market. In a perfected retail gasoline market, the price of midgrade would present an arbitrage opportunity since it is above the price of mixing premium and regular. Consumers could take advantage of this possibility and acquire midgrade cheaply. Even if consumers choose not to mix their own gasoline because of the inconvenience, the apparent arbitrage possibility has not been eliminated by competition between gas stations. At a minimum, midgrade pricing is a robust and durable example of how it may not always be possible to claim that equilibria must be arbitrage-free, because we cannot always expect:

> "that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components"
>
> Modigliani and Miller (1958)

Our data set invites and permits several extensions. Both time series and cross-sectional patterns of overpriced midgrade provide interesting avenues for future research. The clustering of mispricing, the behavior of midgrade mispricing during price wars, and whether nearby competitors attempt to punish defections from midgrade overpricing are all possible extensions of our current research.

There are other markets where linear post-production mixing suggests that we should consider how perfected post-production trading markets may help characterize equilibrium price restrictions or at least frame them as potential anomalies to be examined further. Our two theorems are a starting point for the empirical analysis of relative pricing in such markets. Our pricing critique may apply to other situations where posted prices are not linear. One example is when fast food restaurants charge less for a "value meal" bundle of products than for a subset of the individual items comprising that bundle. Other examples might include such products as tour packages where component and bundle prices are simultaneously quoted.

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Appendix A

OPIS Data Collection Procedures

(taken from http://opisnet.com/methodology.asp, 9/30/07)

Overview

For almost three decades, OPIS has been a news and pricing leader in the downstream refined products marketplace. We have served customers throughout the many industry segments - traders, suppliers, commercial end-users, wholesalers and retailers - with up-to-the minute, award-winning news, analysis and pricing that appears in our many published reports and on-line services.

In that time, OPIS has become the only provider of U.S. spot, rack and retail prices - giving us a complete picture of the marketplace that is rivaled by no other petroleum information supplier.

OPIS editors collectively have more than 175 years experience covering petroleum markets. Our editors know that our numbers are commonly referenced by the industry, but we remain at arms' length. OPIS does not invest in oil companies, speculate on oil prices or accept special favors.

This document explains our methodology for price collection at all levels and the steps we take to ensure data integrity and accuracy.

Anti-Trust Policy

For almost 30 years clients have trusted OPIS to adhere to strict anti-trust guidelines in collecting and distributing sensitive oil pricing data. With oil prices under increasing scrutiny, OPIS recognizes that suppliers cannot afford even the slightest perception of price sharing or price signaling. That's why OPIS does not provide price notification and messaging services for suppliers and embargoes release of all rack pricing data until after the changes become effective to customers.

OPIS Wholesale Rack Pricing

Price Discovery

Every day, including Saturdays, OPIS updates its wholesale terminal prices from hundreds of sources. Some suppliers confirm prices directly using the same pricing messages their customers receive. For many other suppliers, prices are collected from their customers who OPIS deems are reliable sources.

Data Integrity

Verification of prices is done using documents provided by either the supplier or customers. Multiple sources are required for prices received via customer channels. In order for a supplier's price to be added to OPIS rack coverage, their price and the consistent supply of barrels at that location must be verified with multiple customers. It must also be a wholesale terminal rack price and not a commercial or consumer end-user price; this avoids mixing classes of trade and misrepresenting true wholesale postings and averages.

OPIS uses several levels of automation to make sure prices that have not changed at usual intervals are fresh. If a price has not changed in 48 hours, it is electronically flagged and a pricing specialist is alerted so as to track down whether the number still represents an active and meaningful listing. OPIS specialists pinpoint prices that are outside specified reasonable parameters to avoid displaying inactive prices where product may not be available or where special circumstances may dictate that the number is not representative of where most wholesale commerce is taking place. Products tagged as "out-of-product" will not be part of the OPIS lows, highs or averages.

Time Stamp (all times are EST)

9:00 a.m. - OPIS wholesale terminal prices for gasoline, distillate, and other products are updated and ready for release.

10:00 a.m. - OPIS contract summary data used for benchmarking is available. This file is delayed to allow time for further verification to ensure the integrity and accuracy of all the prices before the information is calculated. The contract data includes the Contract Average which is a gross price that OPIS has had since 1995. As of April 1, 2004, we added a Contract Low and Contract High as well as Contract Net Average, Contract Net Low and Contract Net High pricing. Branded and Unbranded numbers are also available as Contract prices. The contract data is frozen for 24 hours to allow customers to reconcile exchanges, sales or other benchmark deals. The contract summary data is also archived. The reason OPIS created the Contract summary data is because OPIS updates price moves throughout the day and publishes them on demand for clients.

6:00 p.m. - OPIS archives the closing rack price database for that business day. The current day's history is available the next business day. The OPIS rack history database is the largest of its kind and dates back to December 15, 1980.

Rack Formats

OPIS Standard Display -- Dates back to 1980 and generally includes one price per supplier for an individual city. OPIS Standard Display selects only one terminal location per supplier based on the location where product is priced the most competitively and where the majority of customers in a particular city lift barrels. This allows the OPIS contract summary data, especially the contract average, which is widely used as a benchmark, to have a consistent methodology and avoid being manipulated by a supplier in a given city. OPIS verifies this data each day to ensure consistency and accuracy in calculating contract averages for benchmark purposes.

OPIS Terminal Display -- Dates back to 1996 when OPIS purchased Computer Petroleum Corporation (CPC). This format includes multiple-supplier listings for individual cities, even if a supplier consistently posts the same price at multiple terminals in a given metropolitan area. OPIS Terminal Display includes all terminal locations for any rack city to provide full supplier coverage.

Rack Pricing History

In addition to providing daily, up-to-the-minute wholesale rack prices, OPIS maintains the largest and most extensive wholesale terminal price historical database of any company in the world. OPIS' historical rack prices date back to 1981, when oil prices were decontrolled. Prices are available on a daily, weekly or monthly basis by market, by company, and by product.

OPIS Retail Gasoline Pricing

Price Discovery

Every day OPIS captures station-specific retail gasoline and diesel prices for up to 120,000 service stations throughout the United States. Through exclusive relationships with credit card companies, direct feeds and other survey methods, OPIS is able to provide the most comprehensive and accurate pump prices in the industry.

The OPIS retail data is relied on by some of the top companies in the country to provide consumers with the most accurate and timely information available including AAA, Microsoft, Mapquest, America Online, Garmin, Verizon, Sprint and many more.

Data Integrity

To ensure accuracy of the retail prices, OPIS scrubs the data through a number of computer programs to make sure the prices are current and are for pump gasoline purchases only - not for in-store purchases that may include non-gasoline products.

OPIS gets prices for most major retailers regardless of whether the station is company operated, jobber owned or dealer operated. Included in the feed are many of the more aggressive c-stores such as WAWA, QuikTrip, Maverik and Sheetz and most of the discount chains and supermarkets such as Wal-Mart, HEB and Kroger.

OPIS has daily, weekly and monthly standard reports as well as customized reports which allow the user to slice and dice the data to get the view of the market they need to make smart decisions. In addition, OPIS has retail history going back as far as 1996 at the station level and can quickly roll the data up to nearly any geographic criteria you desire.

Time Stamp

OPIS is able to capture prices in near real-time - as soon as the swipe happens – at more than 25,000 locations. OPIS Is currently working with the major networks in order to bring you more and more prices as they change and expects a major percentage of the 120,000 stations to be available in real-time by the end of this year.

The stations which currently don't have the ability to be captured in real-time are updated via a batch file each morning and each price has the actual transaction date of the purchase. The daily feed through the batch process has transactions that are from 1-5 days old with the majority of prices being no older than 3 days.